Study of statistical systems in various spatial dimensions based on a fractional quantum mechanics

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Abstract,

The purpose of this work is to study some problems in statistical mechanics based on the fractional classical and quantum mechanics. At first stage we have presented the thermodynamical properties of the classical ideal gas and the system of N classical oscillators. In both cases, the Hamiltonian contains fractional exponents of the phase space (position and momentum). At the second stage, in the context of the fractional quantum mechanics, we have calculated the thermodynamical properties for the black body radiation, studied the Bose-Einstein statistics with the related problem of the condensation and the Fermi-Dirac statistics.

Keywords: fractional calculus, fractional quantum mechanics, grand canonical partition function, thermodynamic quantities, bosons.

Introduction

Since some years, research has intensified and diversified in the calculation of the fractional derivatives. These are applied quickly thereafter, to physics and to engineering. [1].

Recently, [2-3] have applied this concept in quantum physics and developed a new fractional quantum mechanics.

Fractional Schrödinger equation

The fractional Schrödinger equation is a fundamental equation of fractional quantum mechanics.

It has the following form [3]:

$$i\hbar \frac{\partial \Psi(r,t)}{\partial t} = D_\alpha \left(-\hbar^2 \Delta\right)^{\alpha/2} \Psi(r,t) + V(r)\Psi(r,t)$$

$\left(-\hbar^2\Delta\right)^{\alpha/2}$ is the fractional quantum Riesz derivative

D_α is the generalized coefficient

$V(r)$ is the potential energy operator.

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Resultats

We have divided our work into two parts:

Fractional Classical Statistical Mechanics

In the first part we studied some classical fractional systems and we calculated:

1. The thermodynamic properties of classical fractional ideal gas system using the canonical ensemble and we found that all the quantities are related to the parameter unless the system state equation.

$$Z_N = \frac{1}{N!}\left(\frac{2\pi^{D/2}V}{\Gamma\left(\frac{D}{2}\right)h^D \alpha(\beta D_\alpha)^{D/\alpha}}\Gamma\left(\frac{D}{\alpha}\right)\right)^N$$

The internal energy for this gas is $E_C = \frac{D}{\alpha}NKT$

State equation $P = \frac{NKT}{V}$

K is Boltzmann constant ,$\Gamma(x)$ is Gamma function

2. The thermodynamic properties of a classical ideal gas system of fractional oscillators

Fractional quantum system

In the second part we treated the problem of free and trapped quantum gas in the case of the presence of fractional derivative in the Hamiltonian

$$H_{\alpha,\nu} = D_\alpha\left(-\hbar^2\Delta\right)^{\alpha/2} + V(r)$$

and we have :

1- Calculated the grand canonical partition functions of free and trapped quantum gas and we have deducted their thermodynamic properties.

2- we have seen that the trapped gas presents a second-order phase transition at a temperature TC which is dependent fractional parameters a and n and we have found that the critical temperature is independent of the size box.

3- we have shown that the trapped quantum gases are equivalent to the free quantum gas.

4- Calculated the constant of Stephane Boltzmen in the fractional case.

5- Calculated the canonical partition function of a quantum fractional oscilateurs gas.

recover the standard statistical mechanics results when the fractional exponents become integers

Publications

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